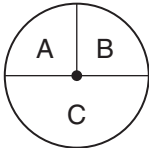
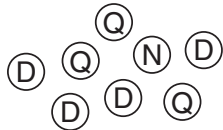
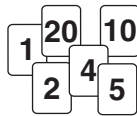


STUDY LINK
7·1

Outcomes and Probabilities



Complete the table.

Experiment	Possible Outcomes	Outcomes Equally Likely?
Example: Spin the spinner. 	A, B, C	No. The area for C is twice as large as each of the other 2 areas.
1. Choose a coin. 		
2. Choose a factor of 20. 		

Use the problems from the table to answer the following questions.

Express each probability as a percent.

- What is the probability of selecting a quarter from the coins in Problem 1? _____
- What is the probability of choosing a factor of 20 from the cards in Problem 2? _____
- Suppose you spin the spinner from the Example in the table. Complete the number sentence below to determine the probability of the spinner landing on A or C.

$$\frac{\quad}{\quad} + \frac{\quad}{\quad} = \frac{\quad}{\quad}$$

Probability of A Probability of C Probability of A or C

Practice

Simplify the expression using the order of operations.

6. $3.8 + 6.4 \div 0.2 - 1.8 * 2.6 - 3.2 \div 0.8$ _____

LESSON
7•1**Carnival Games**

At the carnival, you will play 10 games and will try to win as many prize coupons as possible. You must visit at least three different booths.



<p>Booth 1</p> <p>Two in a Row</p> <p>Flip a coin twice. If the coin lands on the same side both times, you win a prize coupon.</p>	<p>Booth 2</p> <p>Odd Tail Toss</p> <p>Flip a coin once and roll a die once. If you get TAILS and an odd number, you win a prize coupon.</p>
<p>Booth 3</p> <p>Roll It Up</p> <p>Roll a die twice. If the second roll is a greater number than the first, you win a prize coupon.</p>	<p>Booth 4</p> <p>10 or More</p> <p>Roll a die twice. If you get 5 or greater both times, you win a prize coupon.</p>
<p>Booth 5</p> <p>Make the Call</p> <p>Predict the roll of a die. If that number comes up, you win a prize coupon.</p>	<p>Booth 6</p> <p>7 or More</p> <p>Roll a die twice. If the total of the rolls is 7 or greater, you win a prize coupon.</p>



LESSON
7•1

Carnival Games Records



Below, record the number of each booth you visit. Make a tally mark for each prize coupon you win during your 10 games.

Booth Number	Number of Prize Coupons Won
Total Number of Prize Coupons Won	

- Describe a strategy for winning the greatest number of prize coupons in 10 games if you must visit at least 3 different booths.

- At which booths does it seem easy to win?

- Describe how you would change the rules of one game to make it easier to win.

LESSON
7·2**Random-Number Results**

Outcome	Group 1	Group 2	Group 3	Group 4	Group 5	Group 6	Group 7	Group 8	All Groups	% of Total
1										
2										
3										
4										
5										
									Total	100%

STUDY LINK
7·2**Using Random Numbers**

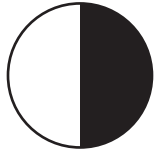
1. A gym teacher is dividing her class into two teams to play soccer. Do you think she should choose the teams at random? _____

Explain. _____

2. The entire school is going to a baseball game. Some seats are better than others. Should the principal select the section where each class will sit at random? _____

Explain. _____

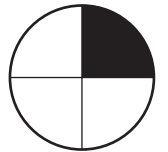
3. The spinner at the right has landed on black 5 times in a row. Renee says, "On the next spin, the spinner is more likely to land on white than on black."



Do you agree or disagree with Renee? _____

Explain. _____

4. The spinner at the right has landed on black 5 times in row. Matthew says, "On the next spin, the spinner has a better chance of landing on white than on black."



Do you agree or disagree with Matthew? _____

Explain. _____

LESSON
7·2

Predict Which Blocks Are in a Bag



1. Pick one person in your group to be the Director.
2. The Director selects 5 blocks and hides them in a bag. The blocks should NOT all be the same color. The group members should NOT see the blocks.
3. Group members take turns drawing one block out of the bag without looking. Each time a block is drawn, group members tally the color.

Example (for first 5 draws):

<i>red</i>	<i>////</i>
<i>blue</i>	<i>/</i>

4. The person who drew the block puts it back into the bag, shakes the bag, and gives it to the next person to draw.
5. After 5 draws, each person writes a prediction for how many blocks of each color are in the bag.
6. Discuss the group's predictions. If everyone has the same prediction, the Director shows the contents of the bag and checks the prediction.
7. If your group does not agree on a prediction, take turns making 5 more draws (for a total of 10). Everyone predicts again and compares predictions.
8. Continue until the group agrees on a prediction. Then the Director shows the contents of the bag.

Repeat this experiment with a different number of blocks in the bag. Try it with 3 blocks. Try it with 7 blocks.

9. Does the number of blocks in the bag make a difference? _____

Explain. _____

10. Do you think there must be a minimum number of draws to make an informed decision about the contents of the bag? _____

Explain. _____

LESSON
7·2**A Table of Random Digits**

This is a table of 500 random digits, which includes the digits 0 through 9. Sometimes statisticians generate random numbers for projects or studies they are conducting by using a random digits table.

9 4 0 1 5 4 6 8 7 4 3 2 4 4 4 4 8 2 7 7 5 9 8 2 0
 9 6 1 6 3 6 4 6 5 4 2 5 8 4 3 4 1 1 4 5 4 2 8 2 0
 7 4 1 0 8 8 8 2 2 2 8 8 5 7 0 7 4 0 1 5 2 5 7 0 4
 9 1 0 3 5 0 1 7 5 5 1 4 7 5 0 4 8 9 6 8 3 8 6 0 3
 6 2 8 8 0 8 7 8 7 3 9 5 1 6 0 5 9 2 2 1 2 2 3 0 4
 9 0 3 1 4 7 2 8 7 7 1 7 3 3 4 3 9 2 8 3 0 4 1 4 9
 1 1 7 4 8 1 2 1 0 2 8 0 5 8 0 4 1 8 6 7 1 7 7 1 0
 5 9 6 2 1 0 6 5 5 4 0 7 8 5 0 7 3 9 5 0 7 9 5 5 2
 1 7 9 4 4 0 5 6 0 0 6 0 4 7 8 0 3 3 4 3 2 5 8 5 2
 5 8 9 0 5 5 7 2 1 6 3 9 6 1 8 4 9 8 5 6 9 9 3 2 6
 6 6 0 6 7 4 2 7 9 2 9 5 0 4 3 5 2 6 8 0 4 6 7 8 0
 5 6 4 8 7 0 9 9 7 1 5 9 4 8 1 3 7 0 0 6 2 2 1 8 6
 5 4 2 4 4 9 1 0 3 0 4 5 5 4 7 7 0 8 1 8 5 9 8 4 9
 9 6 1 6 9 6 1 4 5 9 2 1 6 4 7 8 7 4 1 7 1 7 1 9 8
 3 0 9 4 5 5 7 5 8 9 3 1 7 3 2 5 7 2 6 0 4 7 6 7 0
 0 7 6 5 4 4 6 3 7 6 2 5 3 6 6 9 4 7 4 6 4 9 5 8 0
 6 9 1 7 0 3 7 4 0 3 8 6 9 9 5 9 0 3 0 7 9 4 3 0 4
 7 1 8 0 3 2 6 8 2 5 0 5 5 1 1 1 2 4 5 9 9 1 3 1 4
 0 8 3 4 5 8 8 9 7 5 3 5 8 4 1 8 5 7 7 1 0 8 1 0 5
 5 9 9 8 7 8 7 1 1 2 2 1 4 7 6 1 4 7 1 3 7 1 1 8 1

1. About what percent of the time would you expect each digit to appear? About _____

2. Use the table at the right to make a tally of the digits. Use a calculator to find what percent of the total each digit appears.

3. Are the digits random in the table of 500 digits?

Digit	Tally of Appearances	Number of Appearances	Percent of Total
0			
1			
2			
3			
4			
5			
6			
7			
8			
9			
Total	500	500	100%

STUDY LINK
7•3

Making Organized Lists



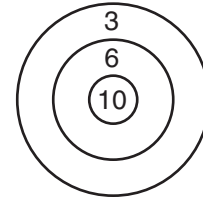
Solve each problem by making an organized list. The list in Problem 1 has been started for you.

1. In how many ways can you make \$0.60 using at least 1 quarter? You can only use quarters, dimes, and nickels.

Q D D D N

Q	D	N
1	3	1

2. You throw three darts and hit the target at the right. List the different total points that are possible.



10 pts	6 pts	3 pts	Total pts

Use what you know about angle measures of sectors to find the probabilities in Problem 3.

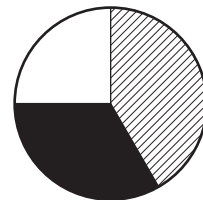
Example:

$$\text{Probability of landing on striped sector} = \frac{150^\circ}{360^\circ} = \frac{5}{12} = 41.67\%$$

3. Find the probability of the spinner landing on

a. white. _____

b. black. _____



LESSON
7•3

Coin-Toss Experiment



- Step 1** Working alone, toss a coin 10 times for Round 1.
Enter a tally mark for each time a HEAD or a TAIL occurs in Round 1.

Coin-Toss Data		
Round	HEADS	TAILS
1		
2		
3		
4		
5		
Totals		

- Step 2** Repeat Step 1 for Rounds 2–5, for a total of 50 tosses.

- Step 3 a.** Record the total number of HEADS and TAILS for your 50 tosses from the frequency table above.

My Totals HEADS $\frac{\quad}{50}$ TAILS $\frac{\quad}{50}$

- b.** Record your partner's HEADS and TAILS totals for all 5 rounds.

My Partner's Totals HEADS $\frac{\quad}{50}$ TAILS $\frac{\quad}{50}$

- c.** Combine your totals with those of your partner.

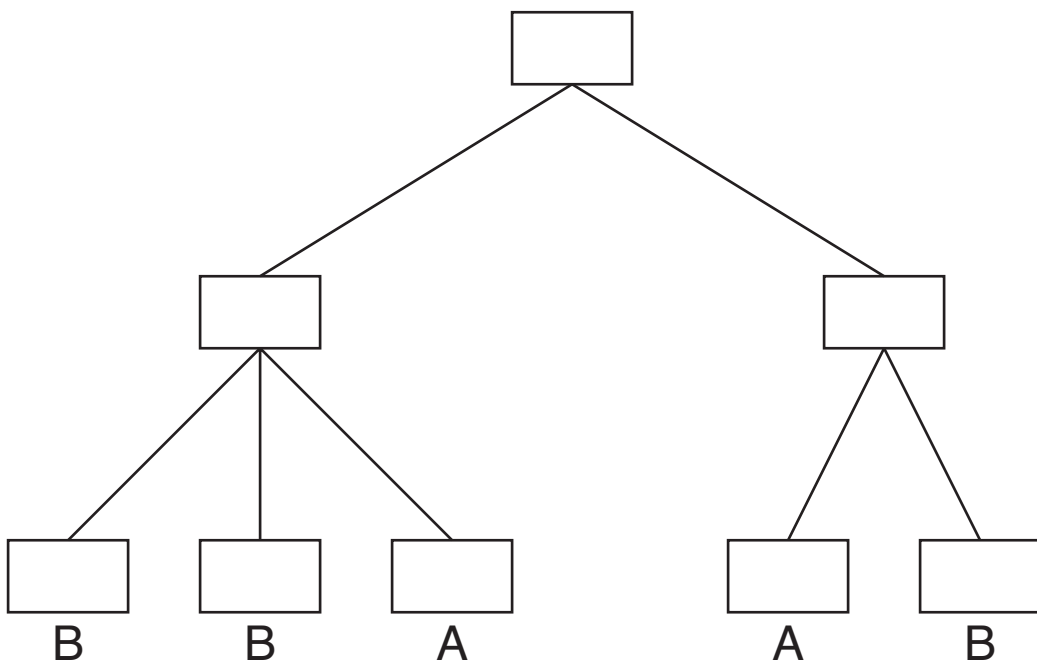
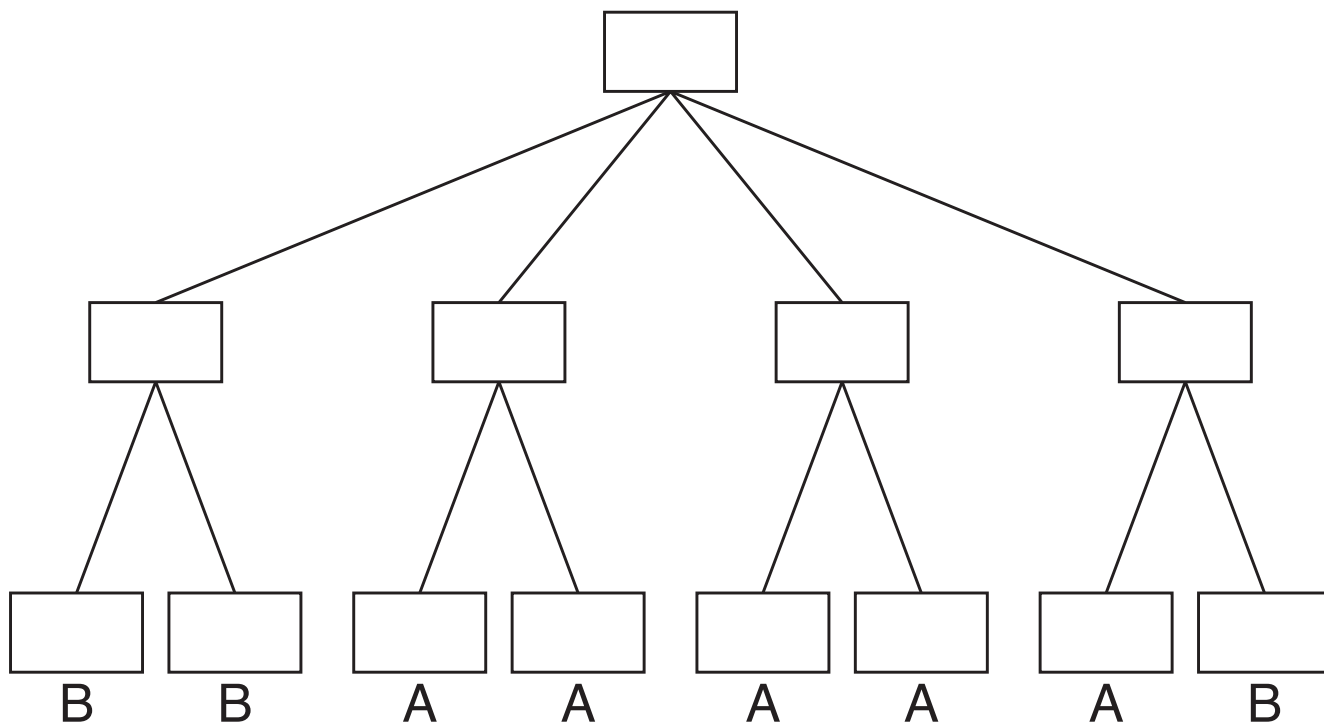
Partnership Totals HEADS $\frac{\quad}{100}$ TAILS $\frac{\quad}{100}$
(Step 3a + Step 3b)

- d.** Now combine your partnership totals with those of the others in your group.

Group Totals HEADS $\frac{\boxed{\quad}}{\boxed{\quad}}$ TAILS $\frac{\boxed{\quad}}{\boxed{\quad}}$
(Step 3c + Step 3d)

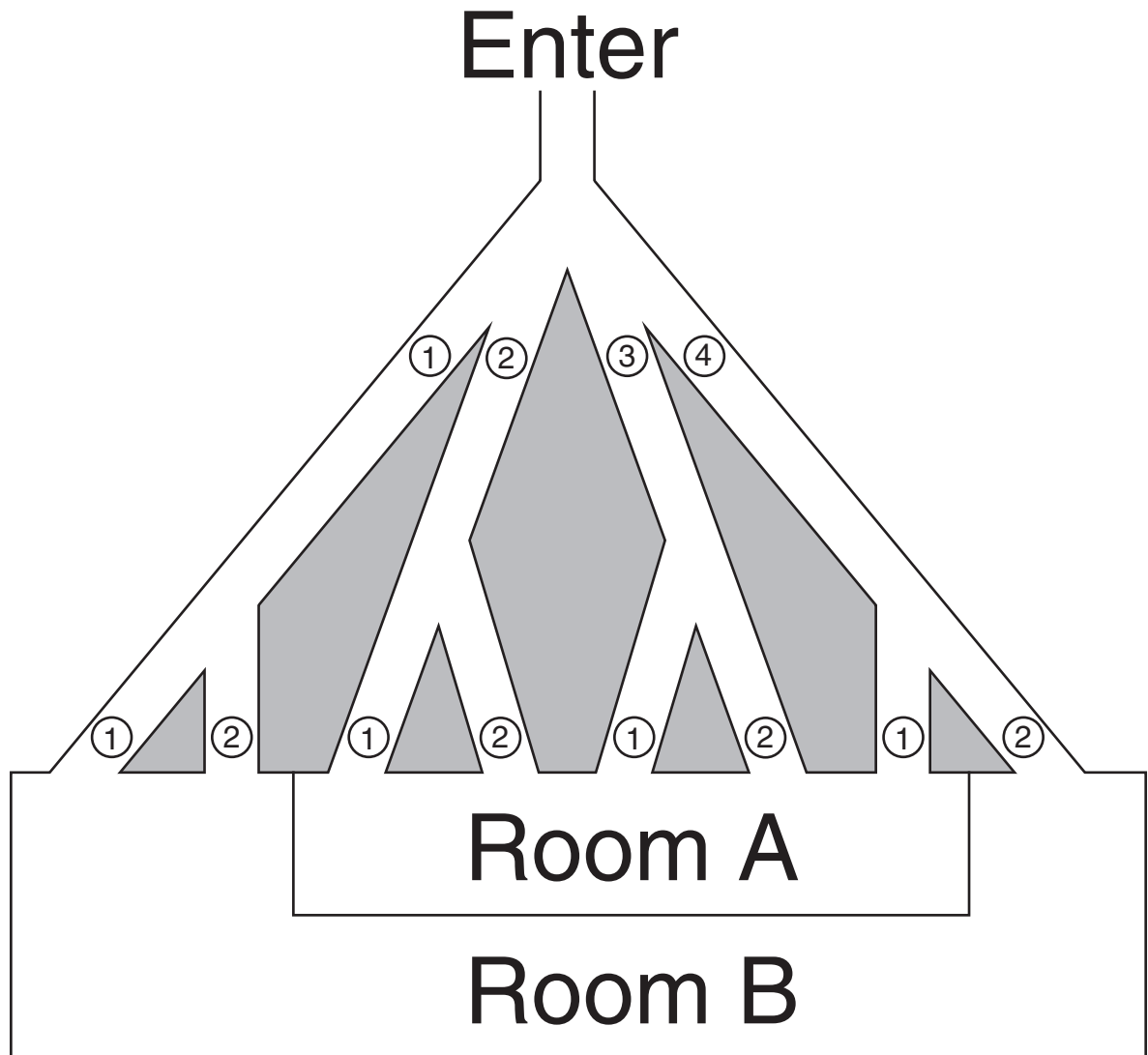
LESSON
7•4

Tree Diagrams



LESSON
7•4

Maze



STUDY LINK
7•4

Lists and Tree Diagrams



Suppose members of the hiking club are served a breakfast bag whenever they have a Saturday morning meeting. Members use the form at the right to place their orders.

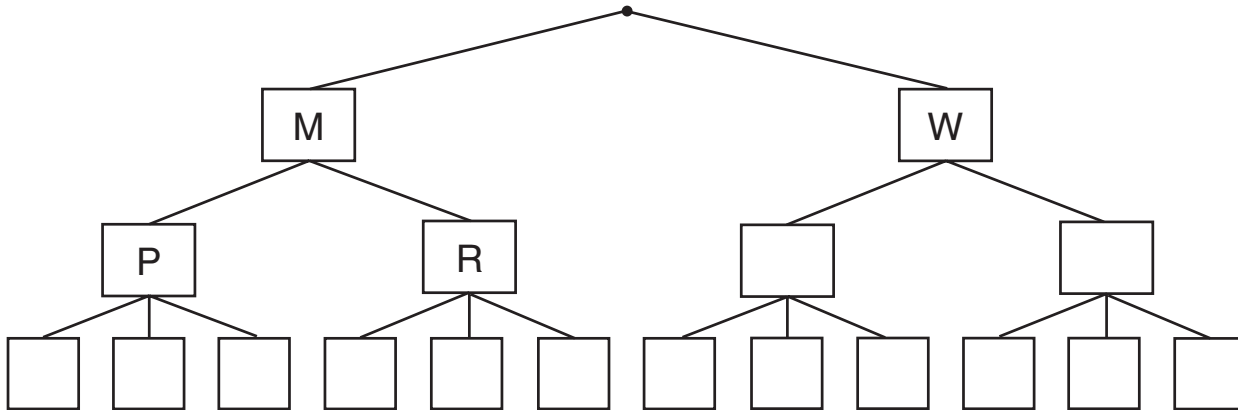
Breakfast Order Form	
Beverage	
<input type="checkbox"/> Milk	<input type="checkbox"/> Water
Bagel	
<input type="checkbox"/> Plain	<input type="checkbox"/> Raisin
Fruit	
<input type="checkbox"/> Apple	<input type="checkbox"/> Banana <input type="checkbox"/> Orange

1. Complete the organized list of the possible breakfast bags.

Beverage	Bagel	Fruit
M	P	A
M	P	B

Beverage	Bagel	Fruit
W	P	A
W	P	B

2. Use your organized list to complete the tree diagram.



3. How many different breakfast bags are possible? _____

4. Suppose 60 members fill out an order form. About how many people would you expect to order milk and a plain bagel? _____ people

5. Suppose each of the 60 members brings 2 guests to the next Saturday meeting. About how many people would you expect to order water, a raisin bagel, and an orange? _____ people

LESSON
7•4

An Amazing Contest



The sixth graders at Bailey School want to raise money to buy a microscope. Students have created the maze shown below, which they will use for a contest. Each contestant pays a fee and tries to go from Start to Exit without retracing any steps. Anyone not ending up at a dead end wins a prize.

The paths at each intersection are numbered. When a contestant reaches an intersection, the contestant chooses the next path at random, using number cards.

Suppose you are going to try the maze. There are 3 different paths at Start. To decide which path to follow, pick a card, without looking, from a set of cards having 1, 2, and 3. If the card you draw is 1, follow Path 1. This leads to a dead end, so you lose.

If the card you choose at Start is 2, follow Path 2. This leads to an intersection that divides into 4 different paths. Pick from a set of cards having 1, 2, 3, and 4 to see which path to follow next. You win if you follow Path 3.

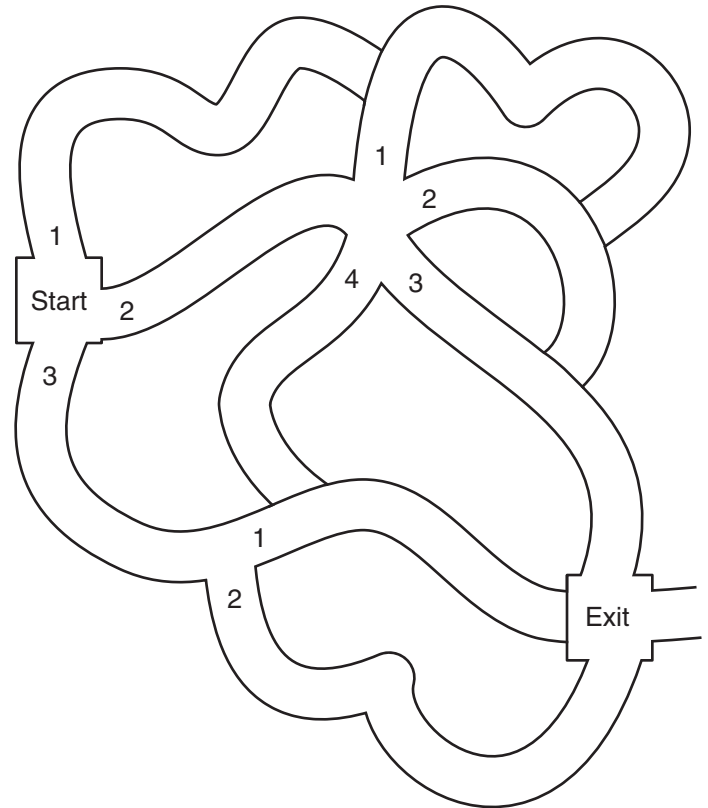
If you choose the number 3 at Start, follow Path 3. This leads to an intersection that divides into 2 different paths. Pick from a set of cards having 1 and 2 to see which path to follow next. You win if you follow Path 2.

Work with a partner. Take turns trying to get through the maze. Each of you should try a total of 6 times.

What fraction of the time did you and your partner reach Exit?

I reached Exit _____ of the time.

My partner reached Exit _____ of the time.



LESSON
7•4**Analyzing the Amazing Contest**

Make a tree diagram of the contest maze to help you solve the following problems.

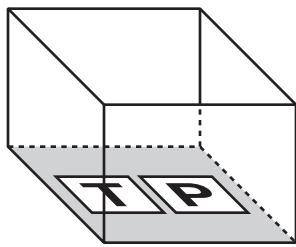
1. If 60 people enter the maze, how many would you expect to reach the exit? _____
2. Suppose the class charges \$5 per person to enter the maze. How much money would the class collect from the contestants? _____
3. If the prize for winning the Amazing Contest is \$12, how much can the class expect to make? _____
4. If the goal for the class is to make \$150, how much should the prize be? _____
5. If the class wants to break even, how much should the prize be? _____
6. Explain how you found the answer to Problem 4.

STUDY LINK
7•5

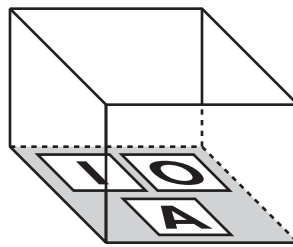
A Random Draw and a Tree Diagram



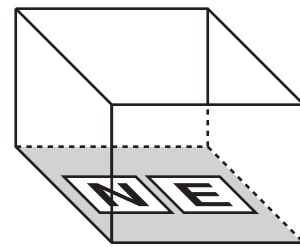
Boxes 1, 2, and 3 contain letter tiles.



Box 1



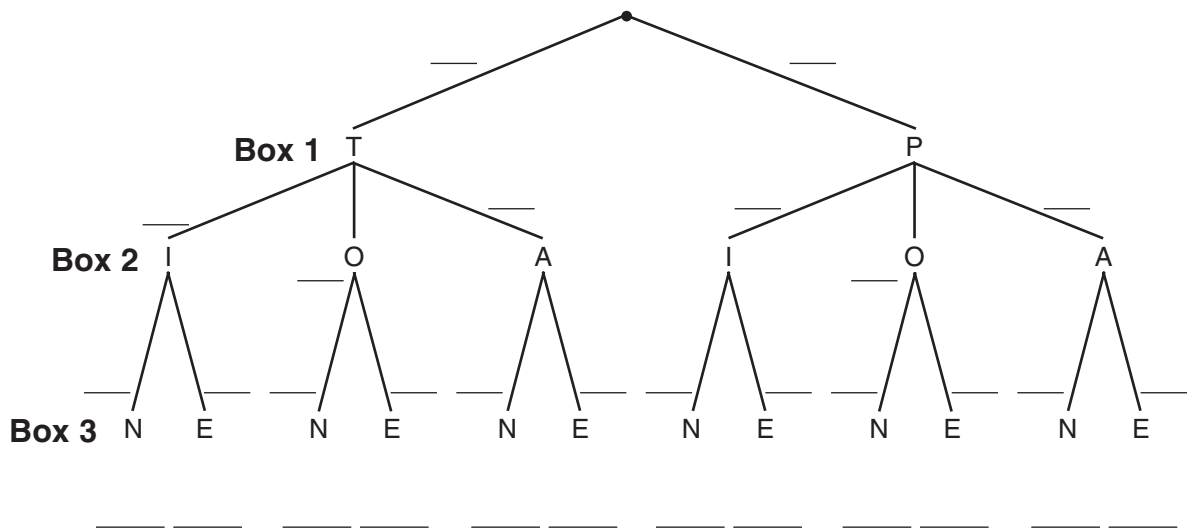
Box 2



Box 3

Suppose you draw one letter from each box without looking. You lay the letters in a row—the Box 1 letter first, the Box 2 letter second, and the Box 3 letter third.

1. Complete the tree diagram. Fill in the blanks to show the probability for each branch.



2. How many possible combinations of letter tiles are there? _____

3. What is the probability of selecting:

a. the letters P and I? _____

b. the letter I, O, or A? _____

c. the letter combinations TO or PO? _____

d. two consonants in a row? _____

Practice

4. $657 \div 18 =$ _____

5. $858.8 \div 38 =$ _____

6. $1,575 \div 125 =$ _____

LESSON
7•5**A Coin-Flipping Experiment**

1. Suppose you flip a coin 3 times.

What is the probability that the coin will land

- a. HEADS 3 times? _____ b. HEADS 2 times and TAILS 1 time? _____
- c. HEADS 1 time and TAILS 2 times? _____ d. TAILS 3 times? _____
- e. with the same side up all 3 times (that is, all HEADS or all TAILS)? _____

Make a tree diagram to help you solve the problems.

2. One trial of an experiment consists of flipping a coin 3 times. Suppose you perform 100 trials. For about how many trials would you expect to get HHH or TTT? _____

What percent of the trials is that? _____

STUDY LINK
7•6

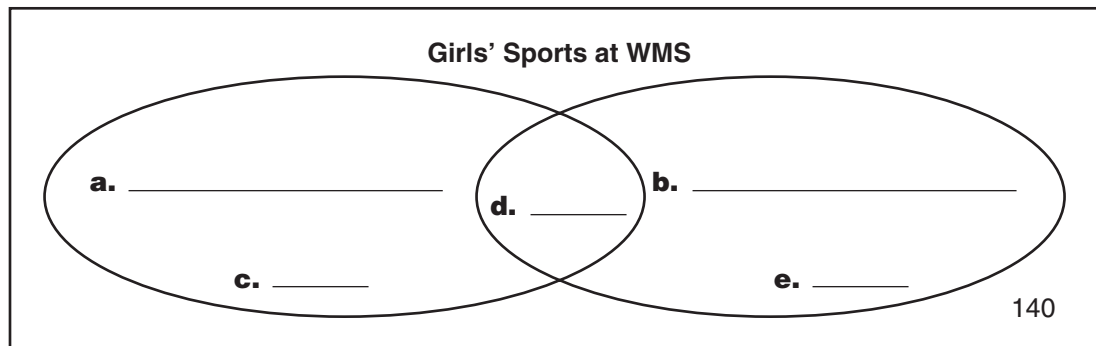
Venn Diagrams



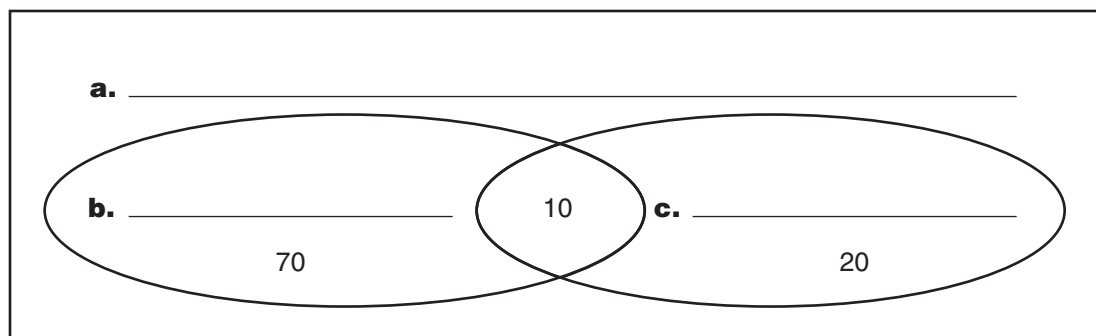
There are 200 girls at Washington Middle School.

- ◆ 30 girls are on the track team.
- ◆ 38 girls are on the basketball team.
- ◆ 8 girls are on both teams.

1. Complete the Venn diagram below to show the number of girls on each team.



- f. How many girls are on one team but not both? _____ girls
- g. How many girls are on the track team but not the basketball team? _____ girls
2. Write a situation (2d) for the Venn diagram below. Complete the diagram by adding a title (2a) and labeling each ring (2b and 2c).



- d. _____
- _____
- _____

Practice

3. $\frac{7}{8} - \frac{9}{20} =$ _____

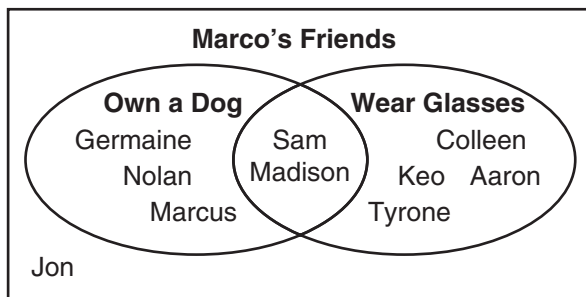
4. $7\frac{1}{3} - 4\frac{5}{12} =$ _____

5. $9\frac{2}{5} - 1\frac{1}{4} =$ _____

LESSON
7•6**Reviewing Venn Diagrams**

A Venn diagram shows how data can belong in more than one group. The diagram is made up of rings that sometimes overlap.

Study the Venn diagram below.



1. Use a yellow highlighter or pencil to outline and lightly shade the Own a Dog ring.

2. List the names of Marco's friends who own a dog.

3. Use a blue highlighter or pencil to outline the Wear Glasses ring.

4. List the names of Marco's friends who wear glasses.

5. Using your blue highlighter, lightly shade the Wear Glasses ring.

a. Which names are in the area of the diagram that is shaded both yellow and blue (green)?

b. What can you tell about the friends whose names appear in the green area of the diagram?

6. Explain why Jon's name is outside the rings of the diagram.

LESSON
7•6

Frequency Tables and Venn Diagrams



Suppose researchers chose 1,000 adults at random and tested them to find out whether they were right- or left-handed. People who showed no preference were classified according to the hand they used more often when writing.

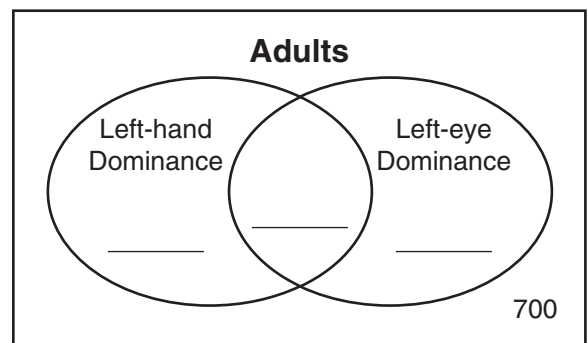
Each person was also tested to determine which eye was dominant.*

Possible results are shown in the table at the right. For example, the table shows that 30 people were left-handed and right-eyed.

Refer to the table to answer the following questions.

		Dominant Hand	
		Left	Right
Dominant Eye	Left	70	200
	Right	30	700

- The sum of the numbers in the table is _____.
- How many people in the sample were right-handed and right-eyed? _____ people
 - How many people were right-handed? _____ people
- How many people in the sample were left-handed and left-eyed? _____ people
 - How many people were left-handed? _____ people
 - How many people were left-eyed? _____ people
- Use your answers from Problem 3 to complete the Venn diagram. Fill in the missing numbers.
- What percent of the people in the sample have their dominant hand and dominant eye on the same side?



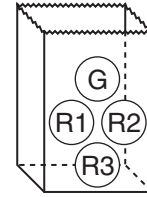
*Eye dominance refers to the tendency to use one eye more than the other in certain tasks involving precise hand-eye coordination and a reasonably distant target. Your dominant eye is the eye you use to aim when you throw darts, for example.

STUDY LINK
7.7

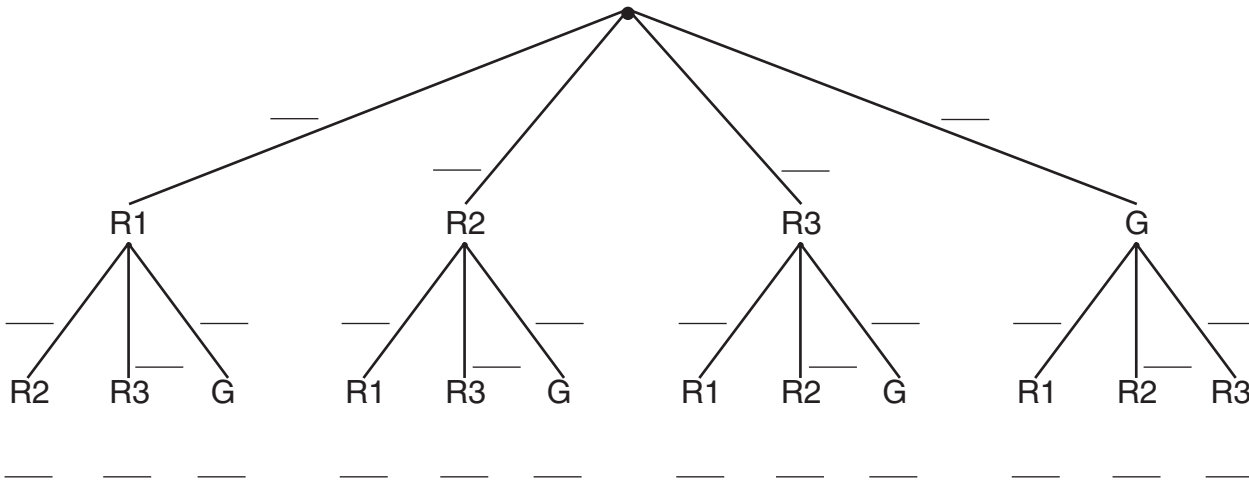
More Tree Diagrams



Denise has 3 red marbles and 1 green marble in a bag. She draws 1 marble at random. Then she draws a second marble without putting the first marble back in the bag.

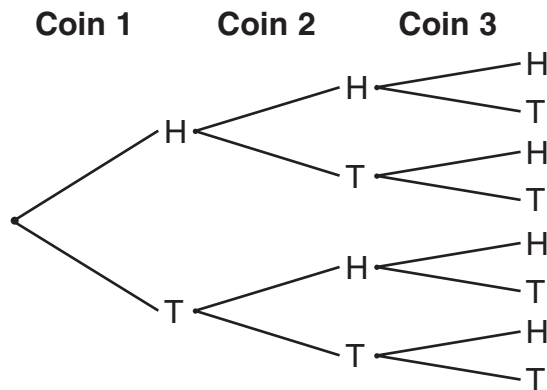


1. Find the probabilities for each branch of the tree diagram below.



- a. What is the probability that Denise will select 2 red marbles? _____%
- b. What is the probability that Denise will first draw a green marble and then a red marble? _____%

2. Three coins are tossed.



Outcomes
HHH
TTT

- a. Complete the table of possible outcomes at the right.
- b. What is the probability of tossing *exactly* 2 HEADS? _____%
- c. What is the probability of tossing *at least* 1 TAIL? _____%

LESSON
7·7**A Coin-Flipping Experiment**

1. Draw a tree diagram to show all possible outcomes when you flip a coin 4 times.

2. How many possible outcomes are there? _____

3. What is the probability that the coin will land
TAILS once and HEADS 3 times? _____

4. What is the probability that the coin will land TAILS
the same number of times it lands HEADS? _____

5. What is the probability that the coin will land on
the same side all 4 times? _____

6. What is the probability that the coin will land
TAILS more often than HEADS? _____

7. What is the probability that the coin will land
TAILS 75 percent of the time? _____

8. What is the probability that the coin will land
HEADS *at least* once? _____

Optional Experiment

9. Do the coin-flipping experiment several times and record the actual results. Combine your results with those of your classmates. Do the actual results come close to the predicted results?

Actual Results _____

Conclusions _____

LESSON
7•7**Making a Fair Game**

Work with your group to figure out how to make the following game fair.

Sum Game

Materials one each of number cards 1, 3, 6, and 10 (from the Everything Math Deck, if available)

Players 1

Directions

1. Mix the cards and place them facedown on the playing surface.
2. Turn over two of the cards.
3. Add the numbers on the two cards. The 1-card (or ace) is worth 1, the 3-card is worth 3, and so on. The sum is your score for the game.
4. You win if you score at least a *certain number* of points. Otherwise, you lose.

Your group's job is to figure out the *certain number* so the game is fair. In other words, you must find the answer to the following question:

What is the least number of points you must score to win half of the time?

Answer: You win if you score at least _____ points.

Explain how you found the answer.

STUDY LINK
7•8

Reviewing Probability



1. Each fraction in the left column below shows the probability of a chance event. Write the letter of the description next to the fraction that represents it.

_____ $\frac{1}{3}$

A. Probability of getting HEADS if you flip a coin

_____ $\frac{1}{4}$

B. Probability of rolling 3 on a 6-sided die

_____ $\frac{1}{2}$

C. Probability of choosing a red ball from a bag containing 2 red balls, 3 white balls, and 1 green ball

_____ $\frac{1}{6}$

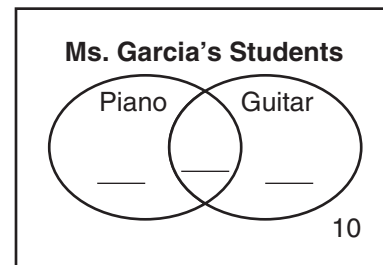
D. Probability of drawing a heart card from a deck of playing cards

2. Sidone bought 3 new swimsuits—1 red suit, 1 blue suit, and 1 white suit. She also bought 2 pairs of beach sandals—1 red pair and 1 white pair. Make a tree diagram in the space at the right to show all possible combinations of swimsuits and sandals.

a. How many different combinations of suits and sandals are there? _____

b. If Sidone chooses a swimsuit and a pair of sandals at random, what is the probability that they will be the same color? _____

3. **a.** Ten students in Ms. Garcia's class play the piano. Seven students play the guitar. Two students play both the piano and the guitar. Complete the Venn diagram at the right.



b. How many students are in Ms. Garcia's class? _____ students

Explain how you know.

LESSON
7•8

Probability and Pascal's Triangle



1. Suppose you toss 2 coins.

- a. Complete the table of possible outcomes at the right.
 b. How many outcomes are possible? _____
 c. Find the probability of tossing

Coin 1	Coin 2
H	H

both HEADS.

only one HEAD.

both TAILS.

2. Look at Row 2 of Pascal's triangle on *Math Masters*, page 240.

- a. What is the sum of the numbers in Row 2? _____
 b. Copy the numbers from Row 2 of the triangle in the spaces below.

- c. Compare your answers for Problems 1b and 1c (above) with your answers for Problems 2a and 2b.

What do you notice? _____

3. Suppose you toss 3 coins. Use Row 3 of Pascal's triangle to complete the following.

- a. How many outcomes are possible? _____
 b. Find the probability of getting

3 HEADS.

2 HEADS and 1 TAIL.

1 HEAD and 2 TAILS.

3 TAILS.

4. Which row of Pascal's triangle would you use to find the possible outcomes and probabilities when 6 coins are tossed? Row _____

5. How many different ways could you answer 5 true/false questions? _____ ways

STUDY LINK
7•9

Unit 8: Family Letter



Rates and Ratios

The next unit is devoted to the study of rates and ratios. Fraction and decimal notation will be used to express rates and ratios and to solve problems.

Ratios compare quantities that have the same unit. These units cancel each other in the comparison, so the resulting ratio has no units. For example, the fraction $\frac{2}{20}$ could mean that 2 out of 20 people in a class got an A on a test or that 20,000 out of 200,000 people voted for a certain candidate in an election.

Another frequent use of ratios is to indicate relative size. For example, a picture in a dictionary drawn to $\frac{1}{10}$ scale means that every length in the picture is $\frac{1}{10}$ the corresponding length in the actual object. Students will use ratios to characterize relative size as they examine map scales and compare geometric figures.

Rates, on the other hand, compare quantities that have different units. For example, rate of travel, or speed, may be expressed in miles per hour (55 mph); food costs may be expressed in cents per ounce (17 cents per ounce) or dollars per pound (\$2.49 per pound).

Easy ratio and rate problems can be solved intuitively by making tables, such as *What's My Rule?* tables. Problems requiring more complicated calculations are best solved by writing and solving proportions. Students will learn to solve proportions by cross multiplication. This method is based on the idea that two fractions are equivalent if the product of the denominator of the first fraction and the numerator of the second fraction is equal to the product of the numerator of the first fraction and the denominator of the second fraction. For example, the fractions $\frac{4}{6}$ and $\frac{6}{9}$ are equivalent because $6 * 6 = 4 * 9$. This method is especially useful because proportions can be used to solve any ratio and rate problem. It will be used extensively in algebra and trigonometry.

$$9 * 4 = 36 \quad \frac{4}{6} = \frac{6}{9} \quad 6 * 6 = 36$$

Students will apply these rate and ratio skills as they explore nutrition guidelines. The class will collect nutrition labels and design balanced meals based on recommended daily allowances of fat, protein, and carbohydrate. You might want to participate by planning a balanced dinner together and by examining food labels while shopping with your child. Your child will also collect and tabulate various kinds of information about your family and your home and then compare the data by converting them to ratios. In a final application lesson, your child will learn about the Golden Ratio—a ratio found in many works of art and architecture.

Vocabulary

Important terms in Unit 8:

Golden Ratio The *ratio* of the length of the long side to the length of the short side of a Golden Rectangle, approximately equal to 1.618 to 1. The Greek letter ϕ (phi) sometimes stands for the Golden Ratio. The Golden Ratio is an irrational number equal to $\frac{1 + \sqrt{5}}{2}$.

***n*-to-1 ratio** A *ratio* of a number to 1. Every ratio $a:b$ can be converted to an *n*-to-1 ratio by dividing a by b . For example, a ratio of 3 to 2 is a ratio of $3 / 2 = 1.5$, or a 1.5-to-1 ratio.

part-to-part ratio A *ratio* that compares a part of a whole to another part of the same whole. For example, *There are 8 boys for every 12 girls* is a part-to-part ratio with a whole of 20 students. Compare to *part-to-whole ratio*.

part-to-whole ratio A *ratio* that compares a part of a whole to the whole. For example, *8 out of 20 students are boys and 12 out of 20 students are girls* are part-to-whole ratios. Compare to *part-to-part ratio*.

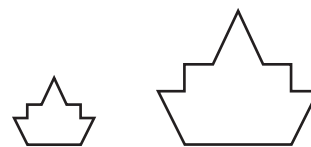
per-unit rate A *rate* with 1 unit of something in the denominator. Per-unit rates tell how many of one thing there are for one unit of another thing. For example, *3 dollars per gallon*, *12 miles per hour*, and *1.6 children per family* are per-unit rates.

proportion A number sentence equating two fractions. Often the fractions in a proportion represent *rates* or *ratios*.

rate A comparison by division of two quantities with different units. For example, traveling 100 miles in 2 hours is an average rate of $\frac{100 \text{ mi}}{2 \text{ hr}}$ or 50 miles per hour. Compare to *ratio*.

ratio A comparison by division of two quantities with the same units. Ratios can be fractions, decimals, percents, or stated in words. Ratios can also be written with a colon between the two numbers being compared. For example, if a team wins 3 games out of 5 games played, the ratio of wins to total games is $\frac{3}{5}$, $3 / 5$, 0.6, 60%, 3 to 5, or 3:5 (read “three to five”). Compare to *rate*.

similar figures Figures that have the same shape, but not necessarily the same size. For example, all squares are similar to one another, and the preimage and image of a *size-change* are similar. The *ratio* of lengths of corresponding parts of similar figures is a *scale* or *size-change factor*. In the example below, the lengths of the sides of the larger polygon are 2 times the lengths of the corresponding sides of the smaller polygon. Compare to congruent.



Similar polygons

size-change factor Same as *scale factor*.

scale factor (1) The *ratio* of lengths on an image and corresponding lengths on a preimage in a *size-change*. Same as *size-change factor*. (2) The *ratio* of lengths in a scale drawing or scale model to the corresponding lengths in the object being drawn or modeled.

Study Link 8•3

1. **a.** \$0.13 per worm **b.** \$3.38
2. a. \$0.18 per oz **b.** \$2.88
3. 150,000 people **4.** 625 gallons
5. \$840; \$15,120 **6.** $\frac{1}{2}$ cent
7. 16 hours; Sample answer: 128 oz = 1 gal;
12 gal = 1,536 oz; $\frac{1,536 \text{ oz}}{1.6 \text{ oz per min}} = 960 \text{ min};$
 $\frac{960 \text{ min}}{60 \text{ min per hour}} = 16 \text{ hr}$

Study Link 8•4

Answers vary.

Study Link 8•5

Answers vary.

Study Link 8•6

1. 25 2. 27 3. 24; 40
4. San Miguel Middle School; Sample answer: I wrote a ratio comparing the number of students to the number of teachers for each school. Richards Middle School, $\frac{14}{1}$; San Miguel, $\frac{13}{1}$.

5.

Shelf	Mystery Books	Adventure Books	Humor Books
1	4	10	18
2	6	15	27
3	8	20	36
4	10	25	45
5	12	30	54
6	14	35	63

- 6.** 14.83 **7.** 88.43 **8.** 12.06

Study Link 8•7

1. 20 2. 57 3. 27 4. 6
5. 250 **6.** 42 **7.** $12\frac{1}{24}$ **8.** $2\frac{8}{9}$
9. $4\frac{11}{20}$ **10.** $3\frac{27}{40}$

Study Link 8•8

Answers vary for 5a and 5b.

- 5. a.** $6\frac{1}{2}$ in.; $4\frac{3}{4}$ in. **b.** 5 in.; 3 in. **c.** $7\frac{1}{4}$ in.; $3\frac{3}{4}$ in.
d. $9\frac{1}{2}$ in.; $4\frac{1}{4}$ in. **e.** 11 in.; $8\frac{1}{2}$ in.
6. Answers vary.
7. Sample answers: **a.** $6\frac{1}{2}$ **b.** 11
8. 2.3 **9.** 57.7 **10.** 10.2

Study Link 8•9

1. **a.** 64 mm **b.** 32 mm
2. a. 45 mm **b.** 180 mm; $\frac{1}{4}$
3. a. 45 mm **b.** 15 mm; 3
4. a. 55 mm **b.** 165 mm; $\frac{1}{3}$

Study Link 8•10

1. **a.** 2:1 **b.** 90° **c.** 9 **d.** 2:1
2. a. 15 **b.** $\frac{3}{2}$ **3.** 90
4. 0.007 **5.** 63.498 **6.** 4.892 **7.** 5.920

Study Link 8•11

1. 1.2; Answers vary.
2. 1.65; No. Sample answer: The ratio for a standard sheet of paper is about 1.3 to 1.
3. Lucille; Sample answer: Compare ratios of correct problems to total problems. Jeffrey's ratio is 0.93 to 1; Lucille's ratio is 0.94 to 1.
4. 12 **5.** 2.8; Answers vary. **6.** 888
7. 21,228 **8.** 15,456 **9.** 126,542

Study Link 8•12

1. **a.** b **b.** e **c.** c
d. a **e.** d
2. a. 40% **b.** 3:5, or $\frac{3}{5}$
3. b. \$7.50 **c.** 8 cans
4. a. 24 members **b.** $\frac{3}{5} = \frac{12}{n}$; 20 free throws
5. Answers vary. **6.** Answers vary.